

# PROBING QCD PHASE DIAGRAM WITH CHARGE FLUCTUATIONS

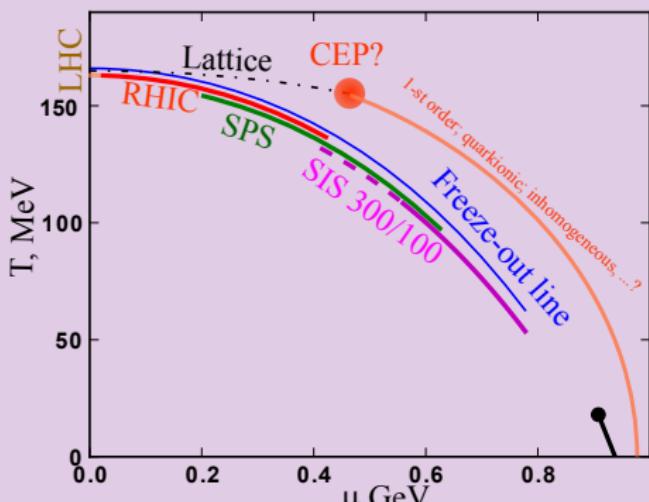
**Vladimir Skokov**  
BNL, nuclear theory group

3 November 2011

# OUTLINE

- Introduction and motivation
- Fluctuations of conserved charges
- O(4) scaling and fluctuations
- Experimental measurement

# QCD PHASE DIAGRAM

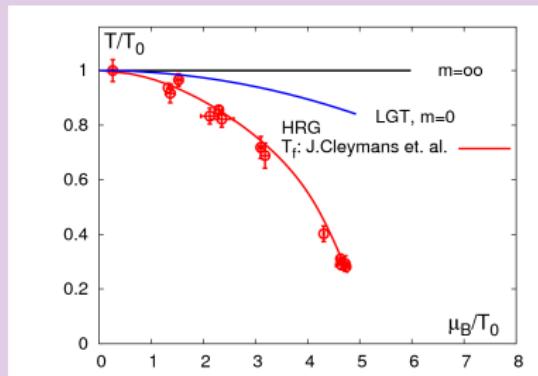


Schematic phase diagram

## Structure of the phase diagram

- crossover at small  $\mu_B$ , underlying O(4) universality class
- (Expected) critical end point, 3d Ising model universality class
- (Expected) first-order transition

# QCD PHASE DIAGRAM



F. Karsch '10

- LGT QCD: curvature of crossover line at small  $\mu_B$  ( $\kappa_q \approx 0.06$ )

$$T/T_c = 1 - \kappa_q \left( \frac{\mu_q}{T} \right)^2 - O(\mu_q^4)$$

$$\approx 1 - 0.0066 \left( \frac{\mu_B}{T} \right)^2$$

- Experiment + HRG model: freeze-out curve

$$T(\mu_B) = a - b\mu_b^2 - c\mu_B^4$$

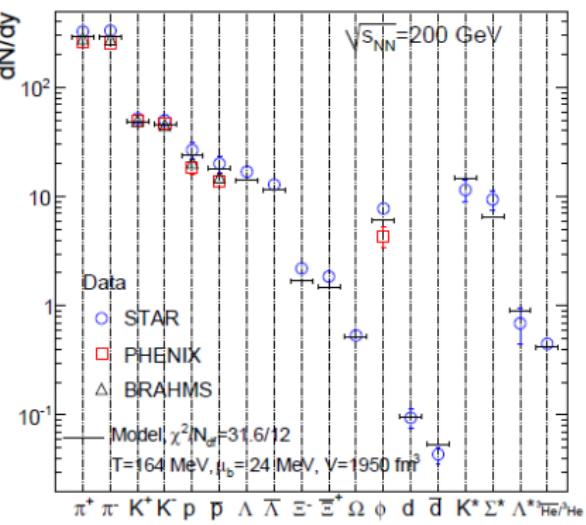
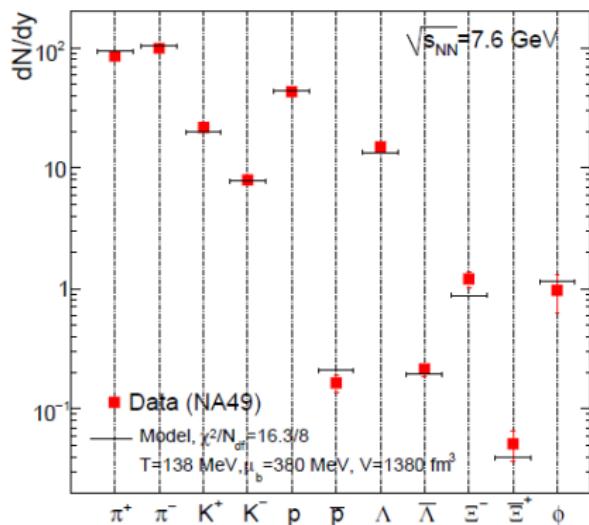
$$\mu_B = d/(1 + e\sqrt{s_{NN}})$$

$$T/T_c \approx 1 - 0.023(\mu_B/T)^2$$

# HADRON RESONANCE GAS MODEL VS EXPERIMENT

**Hadron Resonance Gas (HRG) model:**

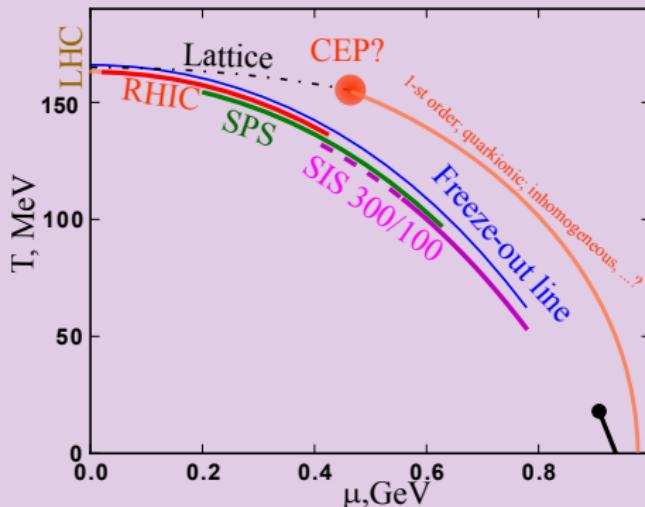
$$p_{\text{HRG}}(T, \mu_B, \mu_S, \mu_Q) = \sum_{\text{mesons}} p_i + \sum_{\text{baryons}} p_i, \quad m_i < 2.5 \text{ GeV}$$



A. Andronic et al.

Particle yields are well described by HRG model

# WHERE IS TRANSITION?

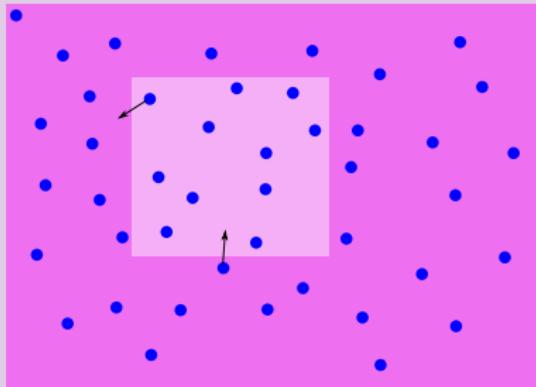


Can we learn something beyond this from particle multiplicities on the freeze-out?

- 1st order phase transition?
- critical end point?
- proximity of freeze-out and crossover line?

# FLUCTUATIONS OF CONSERVED CHARGES: DEFINITION

Assume system in equilibrium with external reservoir with respect to both particle and energy exchange.



Heavy-ion collisions:

- Conserved charges do not fluctuate
- Part of phase space ( $p_t$ -cuts, rapidity cuts, etc)

- Energy density and number of particles = random variables with a certain  $\mathcal{P}$
- $\mathcal{P}(N)$  depends on  $T, \mu$  and  $V$ .
- $\mathcal{P}(N)$  characterises by
  - mean  $\bar{N} \equiv \langle N \rangle = \sum_N N \mathcal{P}(N)$
  - variance  $\sigma^2 = \langle (N - \bar{N})^2 \rangle$
  - higher order moments and/or cumulants  $\chi_n$

$$\text{MGF}(y) = \langle e^{Ny} \rangle = 1 + \bar{N}y + \langle N^2 \rangle y^2 / 2 + \dots$$

$$\text{CGF}(y) = \ln(\langle e^{Ny} \rangle) = \sum_n \frac{c_n}{n!} y^n$$

$$c_1 = \bar{N}, c_2 = \sigma^2$$

$$c_4 = \langle (N - \bar{N})^4 \rangle - 3\langle (N - \bar{N})^2 \rangle^2$$

# FLUCTUATIONS OF CONSERVED CHARGES: DEFINITION

Probability theory:

$$\text{MGF}(y) = \langle e^{Ny} \rangle = 1 + \bar{N}y + \langle N^2 \rangle \frac{y^2}{2} + \dots$$

Thermodynamics, GCE:

$$\text{MGF}(y) = Z_{\text{GC}}(y \equiv \bar{\mu} = \mu/T) = \text{Tr} e^{-\frac{\hat{H}}{T} + \bar{\mu} N}$$

$$\text{CGF}(y) = \ln(\langle e^{Ny} \rangle) = \sum_n \frac{c_n}{n!} y^n$$

$$\text{CGF}(y) = \ln Z_{\text{GC}}(\bar{\mu}) = \frac{V}{T} p(\bar{\mu}) = (VT^3) \cdot \frac{p(\bar{\mu})}{T^4}$$

$$\textcolor{brown}{P}(N)$$

$$Z_{\text{GC}} = \sum_{N=-\infty}^{\infty} Z_C(N) \exp(\mu N)$$

$$\textcolor{brown}{P}(N) = \text{const}(T, \mu) \cdot Z_C(N) e^{\hat{\mu} N}$$

## Experiment:

$\mathcal{P}(N) \sim \langle N^k \rangle = \sum_N N^k P(N) \sim$  cumulants

## Theory:

$p(T, \mu) \sim \partial^n / \partial \mu^n \sim \chi_n \cdot (VT^3) \equiv$  cumulants

# CUMULANTS $\propto$ SUSCEPTIBILITIES

- Fluctuations of net-quark number  $\chi_n^q$  and net-baryon charge  $\chi_n^B$

$$\chi_n^q = \frac{\partial^n(p/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

- Fluctuations of electric charge  $\chi_n^Q$

$$\chi_n^Q = \frac{\partial^n(p/T^4)}{\partial(\mu_Q/T)^n}$$

- Fluctuations of net-strange number...

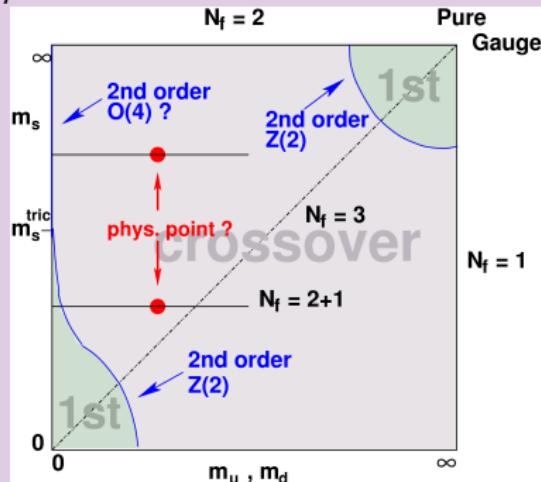
# FLUCTUATIONS OF CONSERVED CHARGES: HRG

**HRG** ( $\mu_S = \mu_Q = 0$ ):

- $p/T^4 = \sum_i f(m_i/T) \cosh(\mu_B/T) + g(T)$
- $\chi_{2n} \propto \cosh(\mu_B/T) \quad \chi_{2n+1} \propto \sinh(\mu_B/T)$
- $\chi_{2n}/\chi_2 = 1 \quad \chi_{2n+1}/\chi_1 = 1$
- $\chi_{2n} > 0$

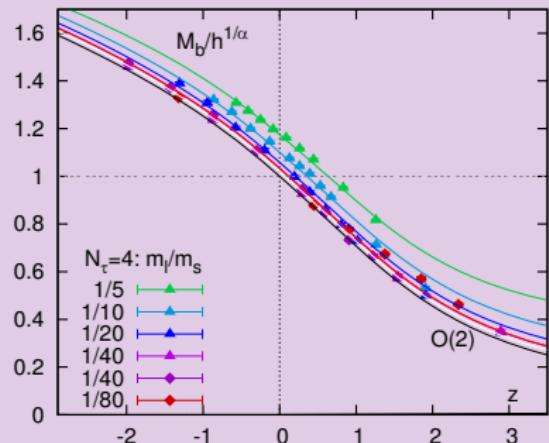
# QCD PHASE DIAGRAM: QUARK MASS DEPENDENCE

$\mu = 0$



- QCD at physical  $m_q$ ?
- $m_l \rightarrow 0$ : O(4) or Z(2)?

F. Karsch et. al. LGT QCD:



- QCD at physical  $m_\pi$  in O(4) scaling
- $N_\tau = 8$  supports these results

# FLUCTUATIONS OF CONSERVED CHARGES: WHY?

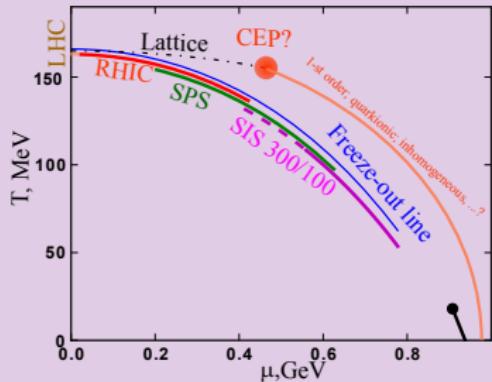
## Properties:

- At CEP: for  $n \geq 2$ ,  $\chi_n \propto \xi^{n\beta\delta/\nu-3} \approx \xi^{5n/2-3}$ , e.g.  $\chi_4 \sim \xi^7$  (M. Stephanov '09)
- Diverging  $\chi_2$  can signal spinodal decomposition of a non-equilibrium 1st order transition (C. Sasaki et. al. '07)

# FLUCTUATIONS OF CONSERVED CHARGES: WHY?

- $m_\pi = 0$  (critical line):  
at  $\mu_B = 0$ ,  $\chi_n \propto \xi^{(n+2\alpha-4)/(2\nu)}$  for even  $n \geq 6$ ,  
e.g.  $\chi_6 \sim \xi^{1.1}$   
at  $\mu_B \neq 0$ ,  $\chi_n \propto \xi^{(n+\alpha-2)/\nu}$  for  $n \geq 3$ ,  
e.g.  $\chi_3^B \sim \xi^{1.1}$
- $m_\pi \neq 0$ : rapid change in crossover region  
(B. Friman et. al. '11)
- Negative values of high order cumulants close to crossover  
(B. Friman et. al. '11)

# CROSSOVER VS CEP



## CEP

- **Good:**  $\chi_4 \sim \xi^7 \sim$  strong signal
- **Bad:** CEP is off from FO line  $\sim$  signal may be washed out by
- **Bad:** low energy of collision  $\sim$  conservation laws dominate the scene (UrQMD, M. Nahrang et. al.)

## Crossover

- **Bad:**  $\chi_6 \sim \xi^{1.1}$
- **Good:** FO line and crossover are close to each other
- **Good:** high energy of collision  $\sim$  reasonable cuts remove impact from cons. laws (Nahrang et. al.)

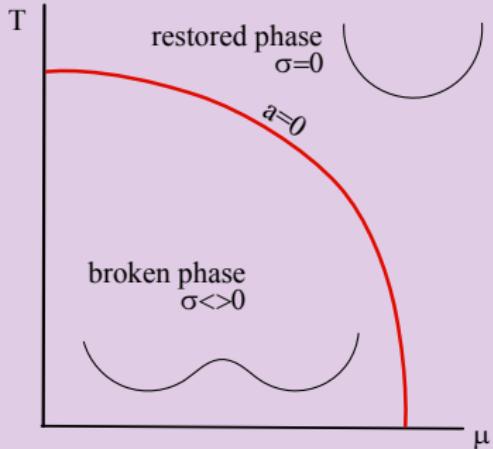
# TOY MEAN-FIELD MODEL: NET-QUARK NUMBER FLUCTUATIONS

Landau theory for 2d-order phase transition ( $m_\pi = 0$ ):

$$\Omega = \frac{a}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4$$

$\sigma$  - order parameter

$$a = \frac{1}{t_0} \left[ \left( \frac{T}{T_c} - 1 \right) + \kappa_q \left( \mu_q/T \right)^2 \right]$$



Minimization of  $\Omega$ :  $\partial\Omega/\partial\sigma=0$  leads to  
 $\sigma_{\min}^2 = -\frac{a}{\lambda}$  for  $a < 0$     and     $\sigma_{\min}^2 = 0$  for  $a > 0$ .

Pressure:  $p = -\Omega(\sigma = \sigma_{\min}) = \frac{a^2}{4\lambda}$

**Second-order cumulant**  $\mu = 0$ :  $\chi_2 \sim (T - T_c)\theta(T - T_c)$

**Higher order cumulants**  $n > 4$   $\chi_n = 0$

# BEYOND MEAN-FIELD

Fluctuations of order parameter  $\sim$  **non-trivial** exponents

Pressure:  $p \sim a^2 \quad \sim p \sim a^{2-\alpha}$

$$a = \frac{1}{t_0} \left[ \left( \frac{T}{T_c} - 1 \right) + \kappa_q \left( \mu_q/T \right)^2 \right]$$

$\alpha$  is non-integer number.

3-dimensional O(4) universality class:  $\alpha \approx -0.21$

$\mu = 0$ : higher cumulants are non-trivial:  $\chi_n \sim (T - T_c)^{-\frac{1}{2}(n-4+2\alpha)}$

$\chi_6 \sim 1/(T - T_c)^{1+\alpha} \quad \chi_8 \sim 1/(T - T_c)^{2+\alpha} \quad$  divergent

$\mu \neq 0$ : higher cumulants are non-trivial:  $\chi_n \sim \left(\frac{\mu}{T}\right)^n a^{-(n-2+\alpha)}$

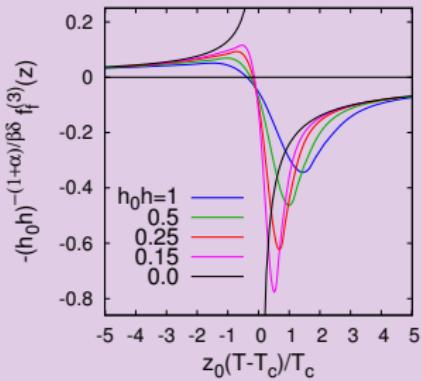
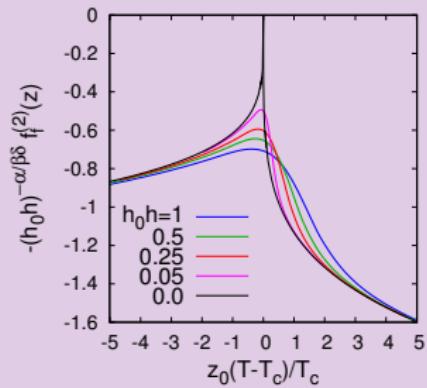
$\chi_4 \sim \left(\frac{\mu}{T}\right)^4 / a^{2+\alpha} \quad \dots$  divergent

# BEYOND MEAN-FIELD: O(4) SCALING FUNCTIONS ON LATTICE

Based on: J. Engels, F. Karsch, arXiv:1105.0584 and  
B. Friman et. al., arXiv:1103.3511

Lattice simulations of O(4) models  $\rightsquigarrow$  singular part of

$$p/T^4 \propto -f(a, h)/T^4, \quad h \propto m_q$$



$$\chi_4(\mu = 0)$$

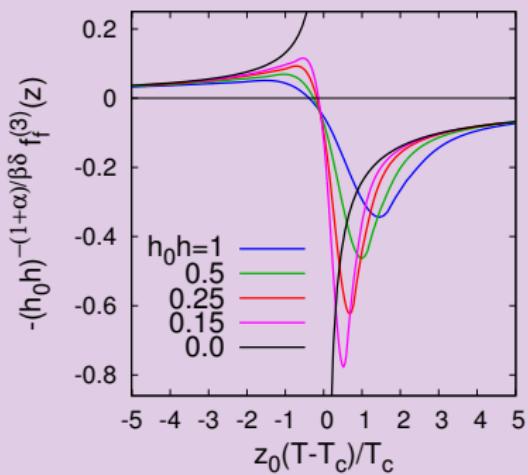
$$\chi_6(\mu = 0)$$

**Does singular part dominates in QCD?**

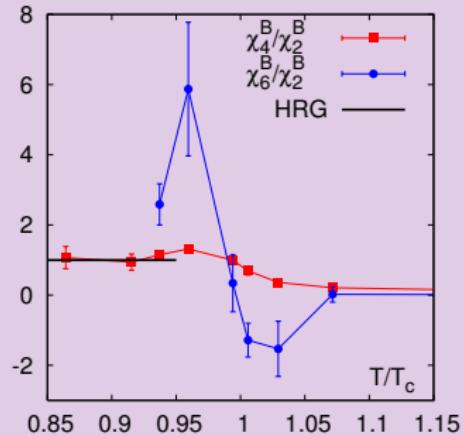
# BEYOND MEAN-FIELD: O(4) SCALING FUNCTIONS ON LATTICE

$$\chi_6(\mu = 0)$$

O(4) scaling:



Lattice QCD:



C. Schmidt, 2010

# MODELING QCD

## Lattice QCD restrictions

- continuum limit for cumulants
- non-zero chemical potential

## QCD inspired model

- O(4) symmetry in limit of vanishing mass for light quarks
- simulation of confinement properties (ratios of cumulants are sensitive to degrees of freedom)

## Polyakov loop-extended NJL or QM model

- O(4) symmetry
- quark interaction with Polyakov loops  $\leadsto$  statistical confinement
- in O(4) scaling regime for physical pion mass, as QCD

# MODELING QCD

Mean-field approximation ( $\alpha = 0$ )?

NO!

Importance of **non-trivial** critical exponents.

- Mean-field,  $m_\pi = 0, \mu = 0, T = T_c$ :

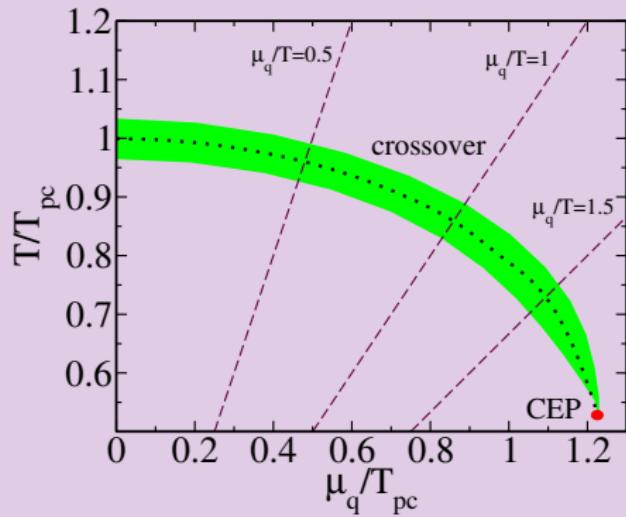
$$\chi_6^{\text{sing}} = 0$$

- Beyond mean-field:

$$\chi_6^{\text{sing}} = \infty$$

- accounts for universal critical behaviour near chiral transition
- reproduces scaling properties and critical exponents  
(Berges '00, B. Stokic et. al. '10)
- respects symmetries  
(Goldstone theorem fulfilled, second-order phase transition in  $O(4)$  model)

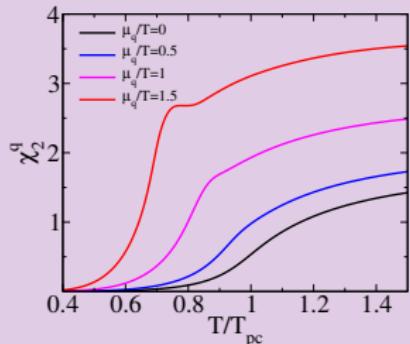
# PHASE DIAGRAM IN FRG PQM



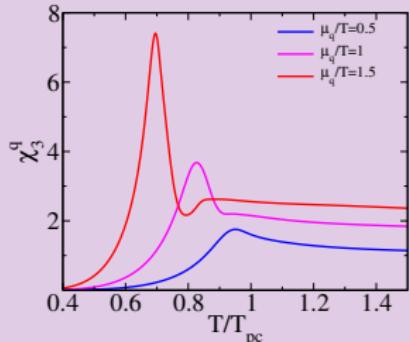
**Crossover:**  $|\partial\sigma/\partial T| > 0.95 \cdot \max(|\partial\sigma/\partial T|)$

# NET-QUARK NUMBER DENSITY FLUCTUATIONS $\delta N_q = N_q - \langle N_q \rangle$

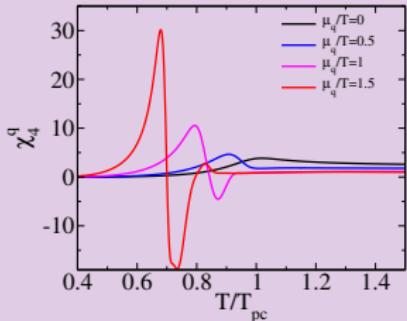
$$\chi_2^q = \frac{1}{VT^3} \langle (\delta N_q)^2 \rangle \rightarrow \frac{N_c N_f}{3} \left[ 1 + \frac{3}{\pi^2} \left( \mu_q/T \right)^2 \right]$$



$$\chi_3^q = \frac{1}{VT^3} \langle (\delta N_q)^3 \rangle \rightarrow \frac{2N_c N_f}{\pi^2} \left( \mu_q/T \right)$$



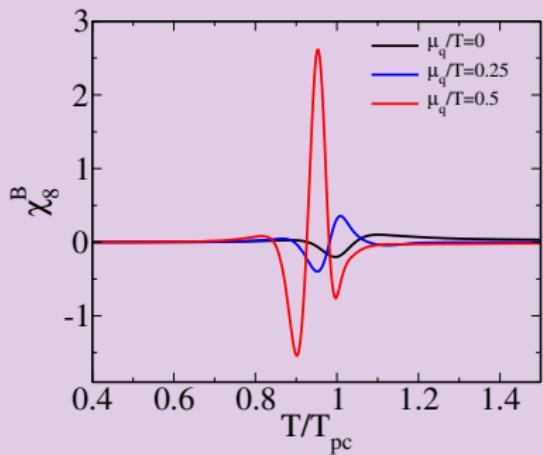
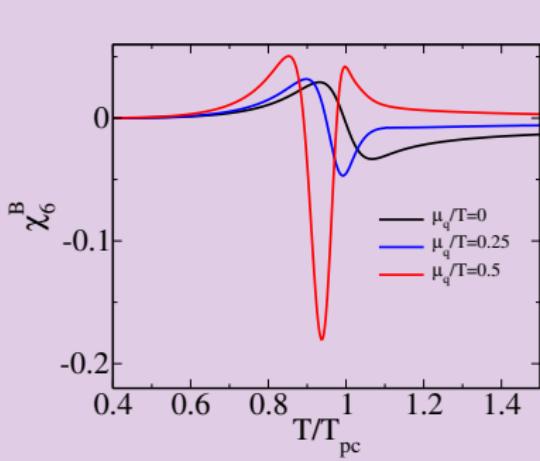
$$\chi_4^q = \frac{1}{VT^3} \left( \langle (\delta N_q)^4 \rangle - 3 \langle (\delta N_q)^2 \rangle^2 \right) \rightarrow \frac{2N_c N_f}{\pi^2} :$$



V.S., B. Friman and K. Redlich PRC'11

- $\chi_2^q$ : non-monotonic structure (diverges at CEP)
- $\chi_4^q$ : **negative** for nonzero  $\mu_q$

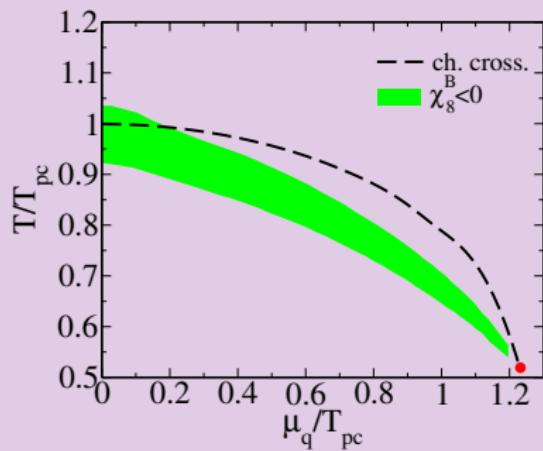
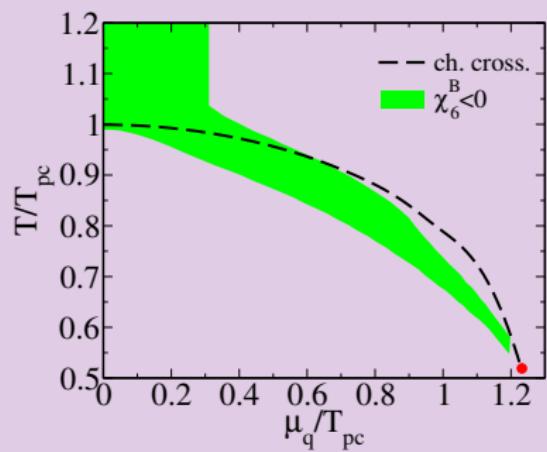
# HIGH-ORDER CUMULANTS OF THE BARYON NUMBER DENSITY



- Negative also at  $\mu_q = 0$
- Temperature range of negative cumulants correlates with crossover temperature
- Many other constraints from O(4) scaling: B. Friman et. al. '11

# HIGH-ORDER BARYON CUMULANT

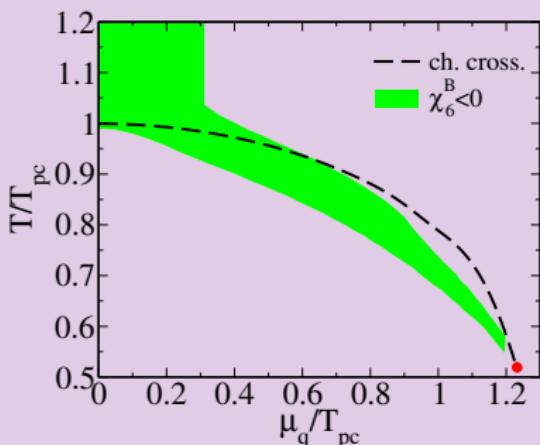
Temperature interval of negative cumulants closest to hadronic phase:



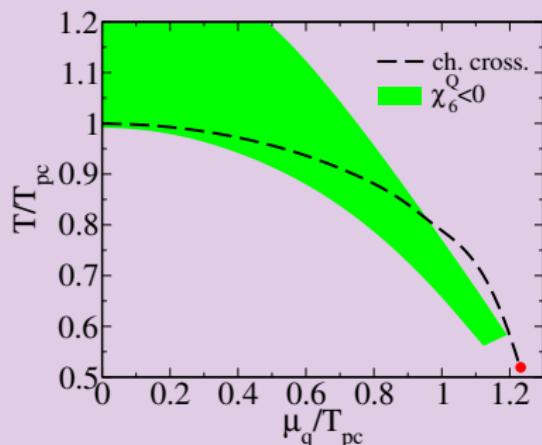
- Negative values (in broken phase!) of high-order cumulants: indicates proximity of freeze-out to crossover
- Accessible experimentally

# ELECTRIC CHARGE FLUCTUATIONS

Baryon charge:

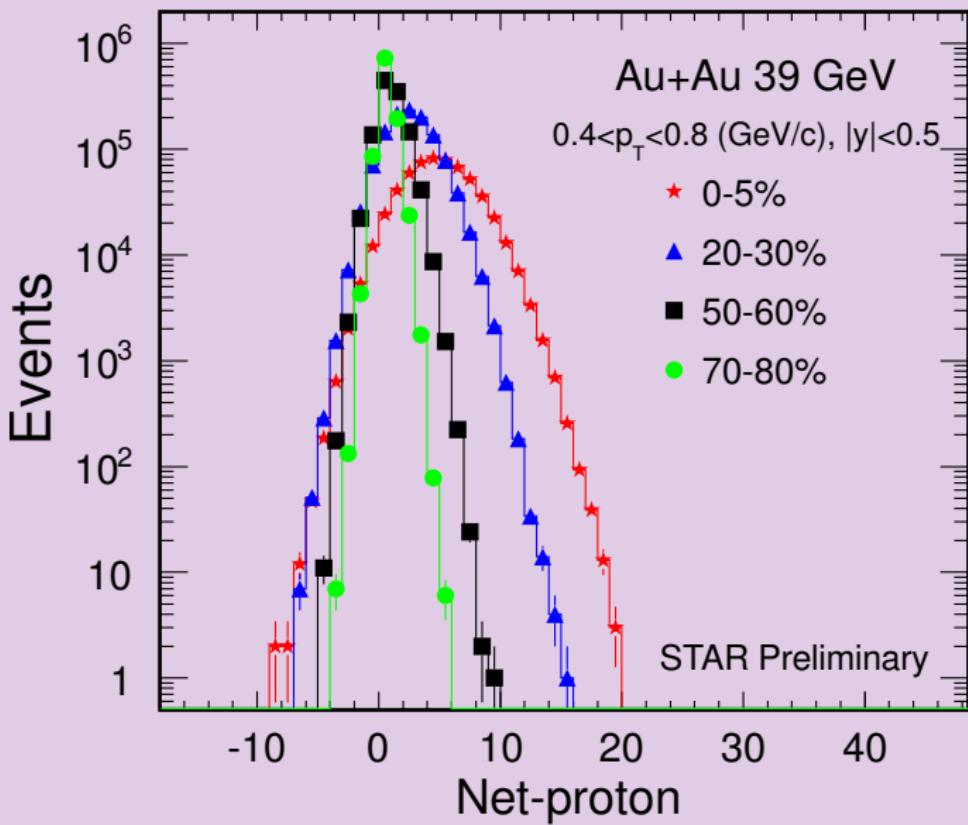


Electric charge:



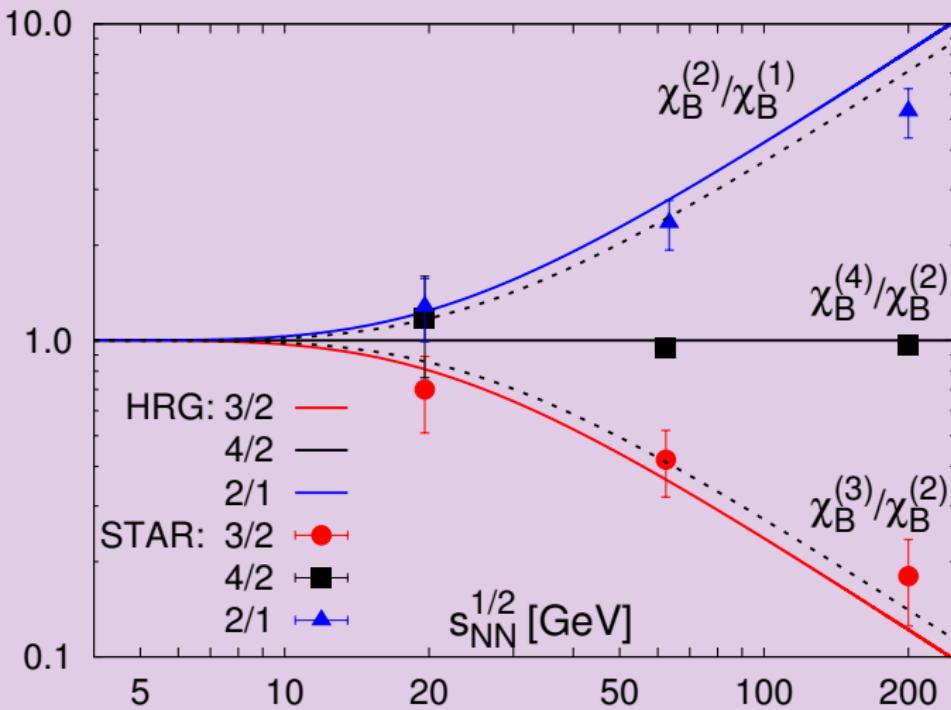
Electric charge fluctuations follow similar pattern as baryon fluctuations

# EXPERIMENT



Xiaofeng Luo, '11

# COMPARISON OF THE HRG MODEL WITH EXPERIMENT



F. Karsch and K. Redlich, '10

## HRG PROBABILITIES

Probability distribution for HRG (Skellam distribution):

$$\mathcal{P}(N) = \left( \frac{N_b}{N_{\bar{b}}} \right)^{N/2} I_N \left( 2 \sqrt{N_b N_{\bar{b}}} \right) \exp [-(N_b + N_{\bar{b}})]$$

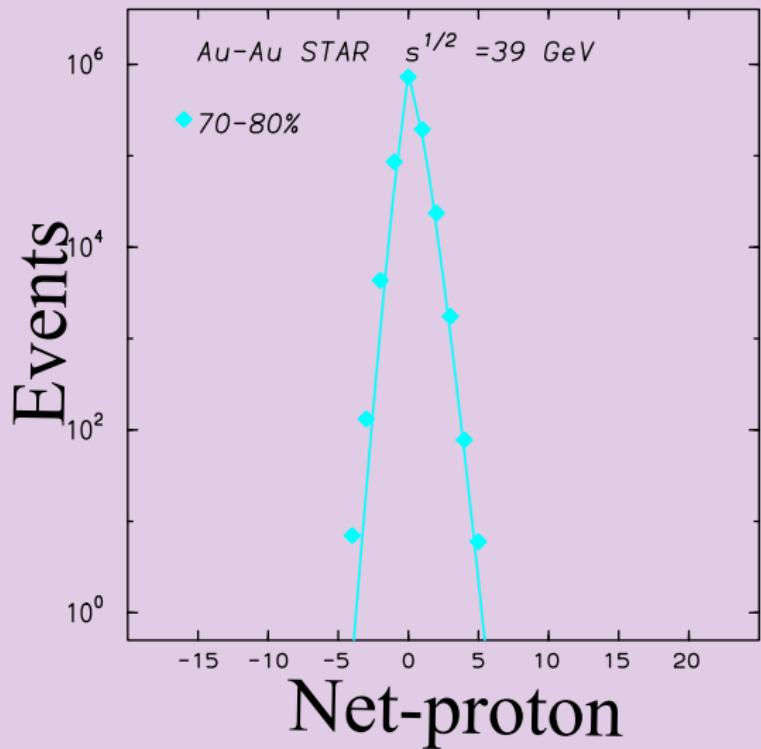
$N_b$  is mean number of baryons (protons)

$N_{\bar{b}}$  is mean number of anti-baryons (anti-protons)

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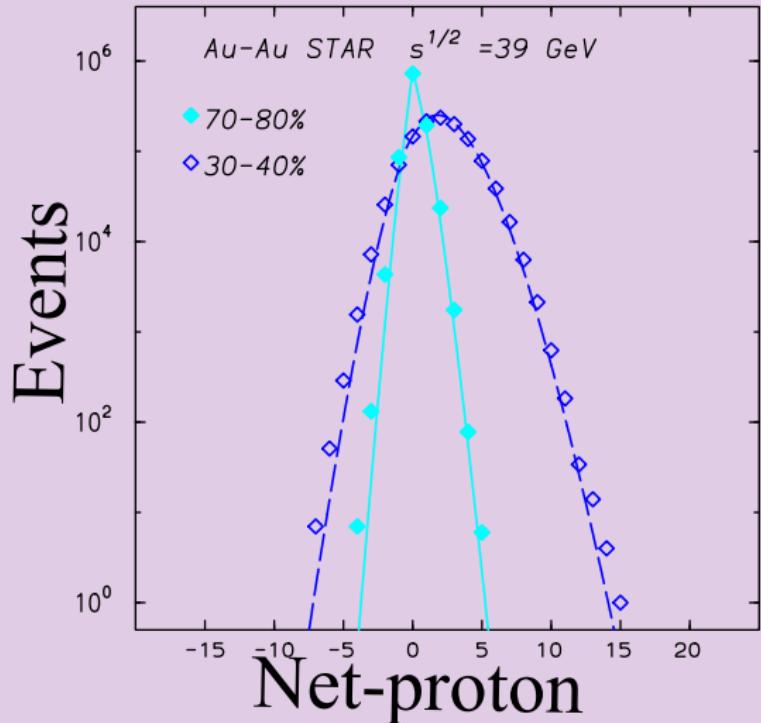
For double charge particles and in case of quantum statistics  $\mathcal{P}(N)$  is to be corrected

# HRG PROBABILITIES



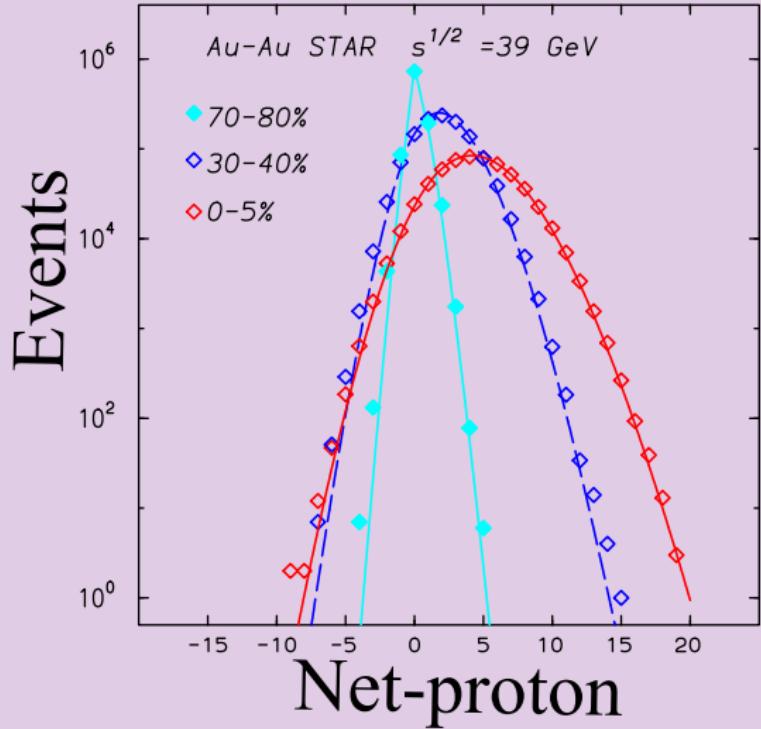
Experimental data: **STAR Preliminary**, Xiaofeng Luo, '11  
HRG line:  $N_p$  and  $N_{\bar{p}}$  are corrected for efficiency

# HRG PROBABILITIES



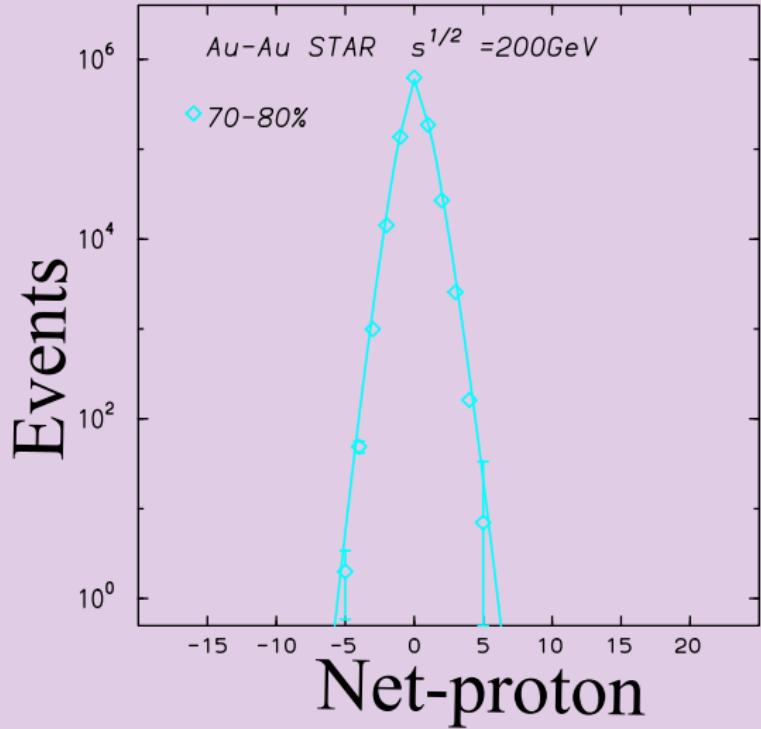
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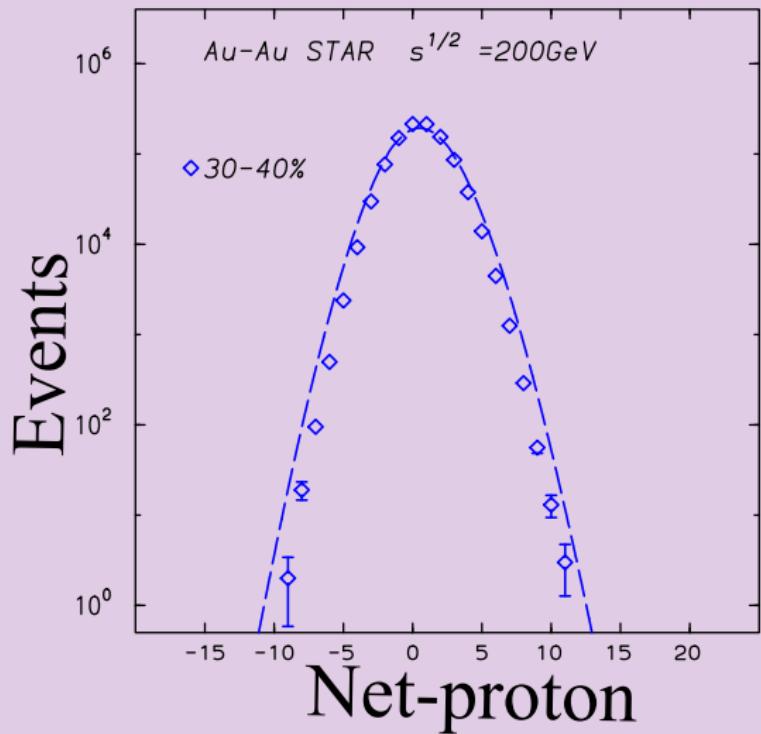
# HRG PROBABILITIES



arXiv:1107.4267

HRG line:  $N_p$  and  $N_{\bar{p}}$  are corrected for efficiency

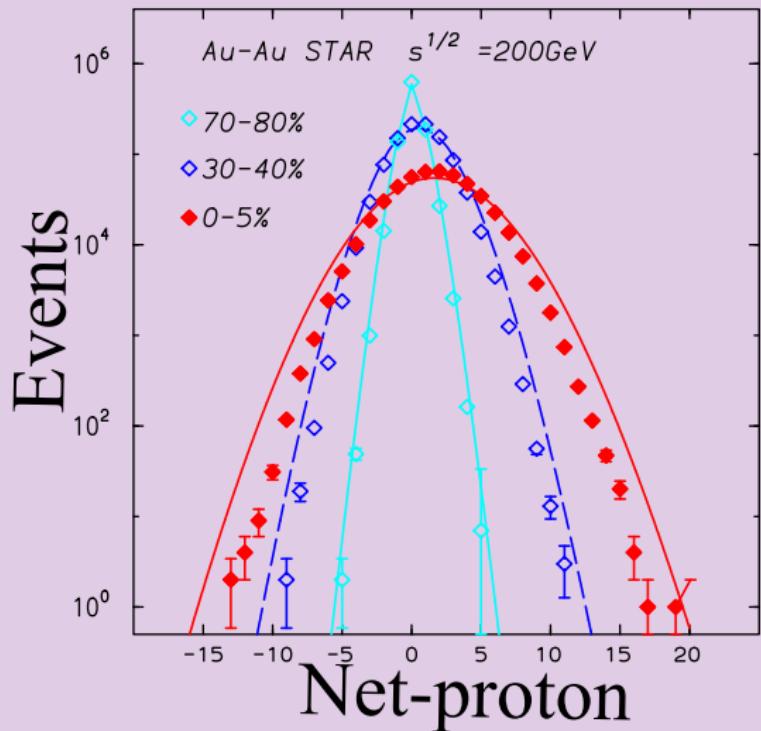
# HRG PROBABILITIES



arXiv:1107.4267

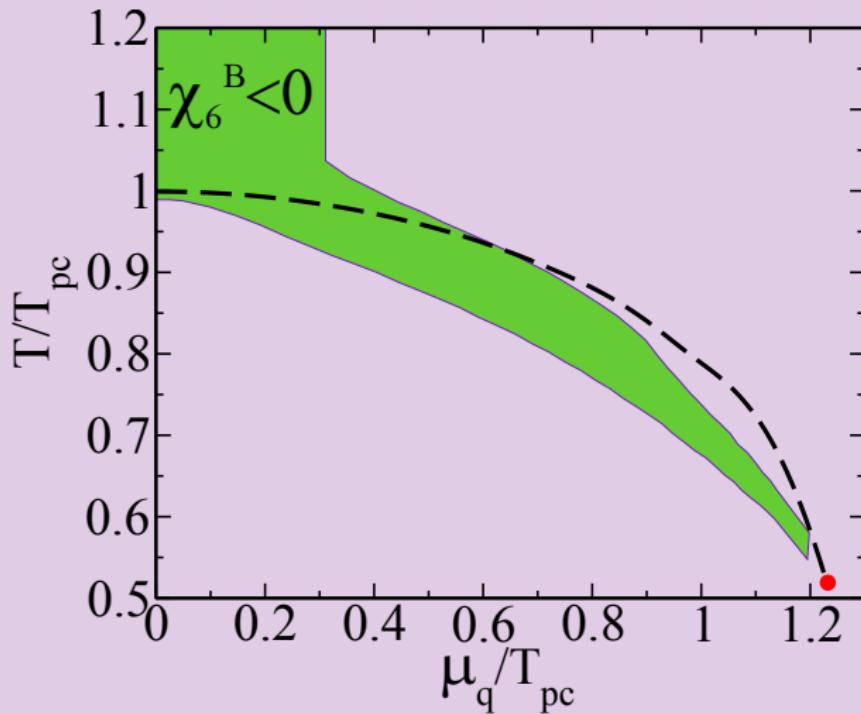
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# HRG PROBABILITIES

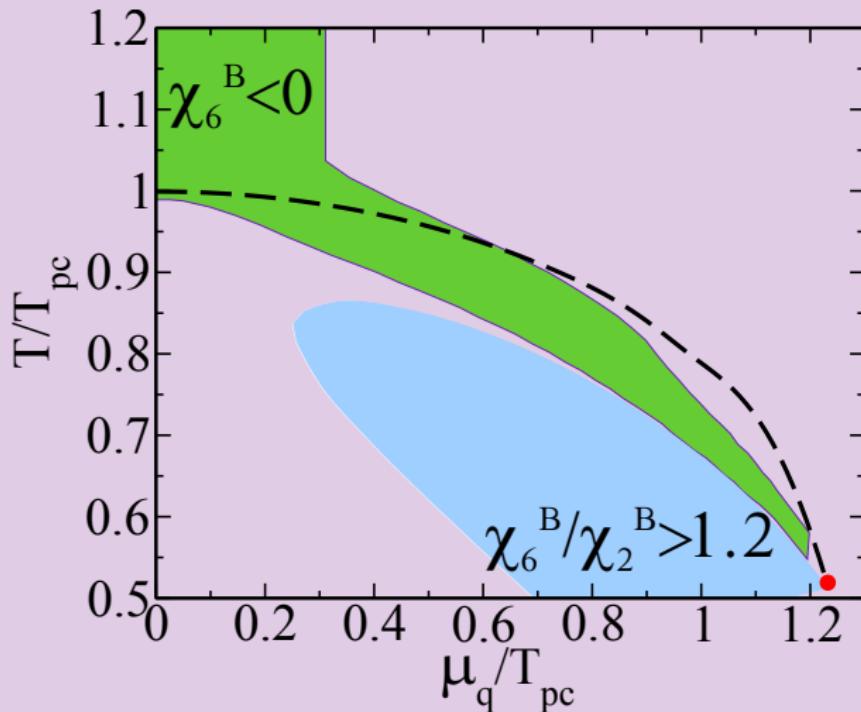


Conclusions are premature  $\sim$  efficiency

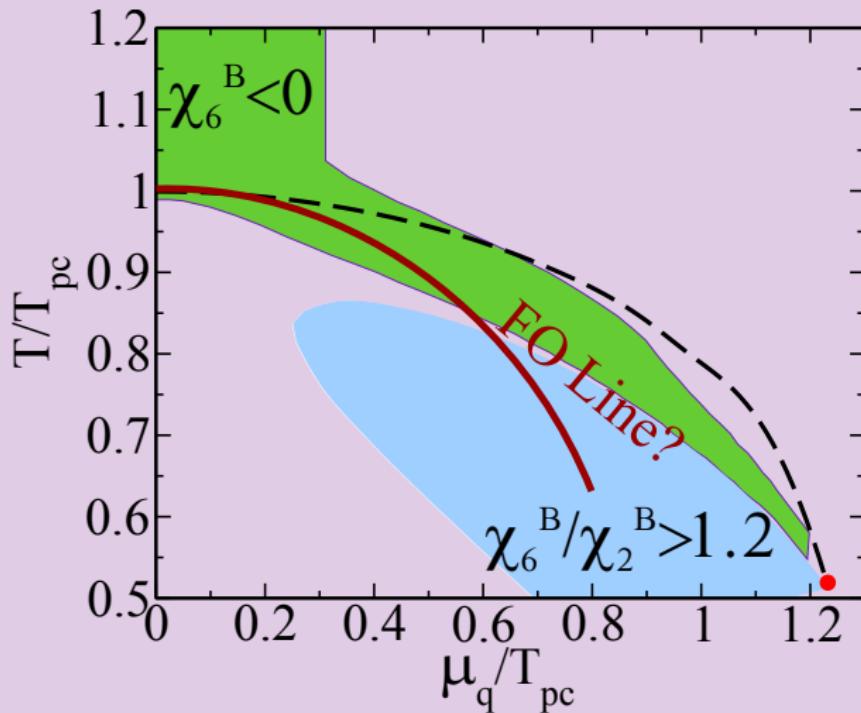
# FO SCENARIOS



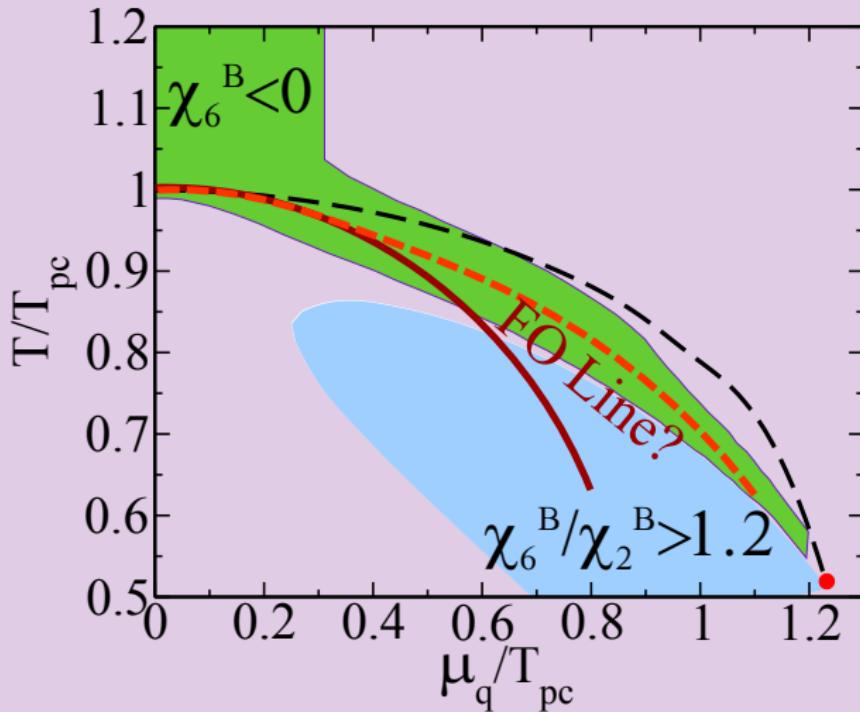
# FO SCENARIOS



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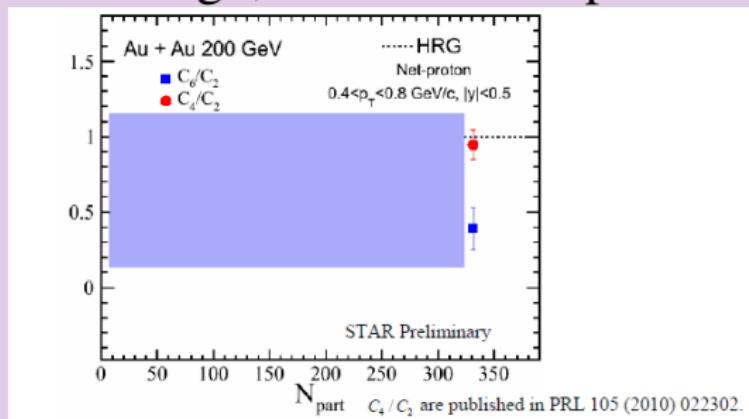
# FO SCENARIOS



- Models are unable relate FO line and PT line
- Low energies: cumulants might be affected by conservation laws

# PRELIMINARY DATA FOR $\chi_6/\chi_2$

L. Chen, one month ago, BNL workshop 2011:



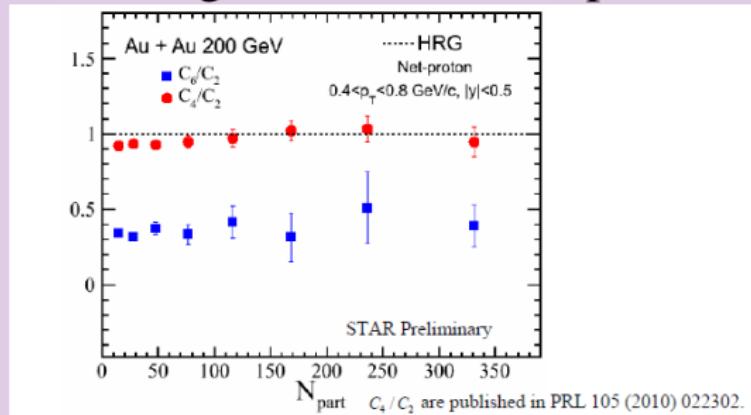
**Red points:**  $\chi_4/\chi_2$ .

Reminder:  $\chi_4/\chi_2$  is not influenced by O(4) criticality

**Blue points:**  $\chi_6/\chi_2$ . Not negative, but suppressed!

# PRELIMINARY DATA FOR $\chi_6/\chi_2$

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**Red points:**  $\chi_4/\chi_2$ . Reminder:  $\chi_4/\chi_2$  is not influenced by O(4) criticality

**Red points:**  $\chi_6/\chi_2$ . Not negative, but suppressed!  
Centrality dependence?

# Thank you for attention

Collaborators: B. Friman, F. Karsch and K. Redlich

# CONCLUSIONS

- **Fluctuations** of conserved charges are **sensitive** probes of the phase structure. They carry information about deconfinement and chiral phase transitions. Fluctuations can be used to identify order and universality class of a phase transition.
- The **negative** values of  $\chi_6^B$  may indicate the proximity of the chemical freeze-out to the crossover line.
- Experimentally, deviations from HRG values of the cumulants or the corresponding probabilities may signal remnants of the transition.
- At the moment, the published data is not conclusive.

# KURTOSIS OF NET-QUARK NUMBER DENSITY

$$\text{Kurtosis } R_{4,2}^q = \frac{\chi_4^q}{\chi_2^q} = \frac{\langle (\delta N_q)^4 \rangle}{\langle (\delta N_q)^2 \rangle} - 3 \langle (\delta N_q)^2 \rangle$$

(S. Ejiri, F. Karsch and K. Redlich '05):

quark content of effective degrees of freedom that carry baryon number

- **Low temperature phase:** dominance of effective three-quark states:

$$P_{\text{baryons}}/T^4 \approx \sum_i F(m_i/T) \cosh(3\mu_q/T)$$

$$\leadsto R_{4,2}^q = 9$$

- **High-temperature phase:**

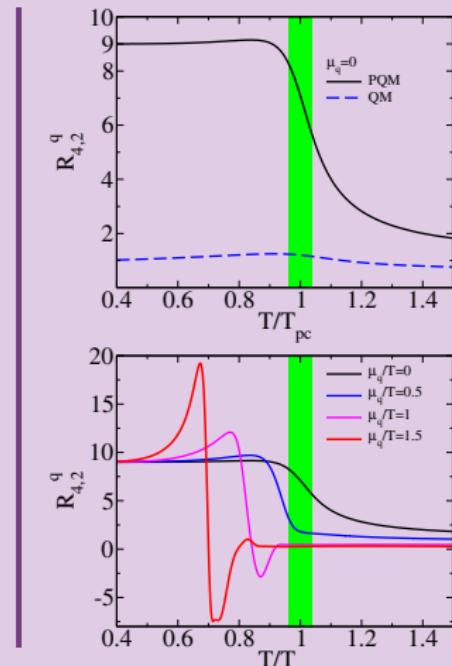
$$P_{\bar{q}\bar{q}}/T^4 \approx N_f N_c \left[ \frac{1}{12\pi^2} \left( \frac{\mu_q}{T} \right)^4 + \frac{1}{6} \left( \frac{\mu_q}{T} \right)^2 + \frac{7\pi^2}{180} \right]$$

$$\leadsto R_{4,2}^q = (6/\pi^2) \approx 1$$

- *PQM: statistical confinement*

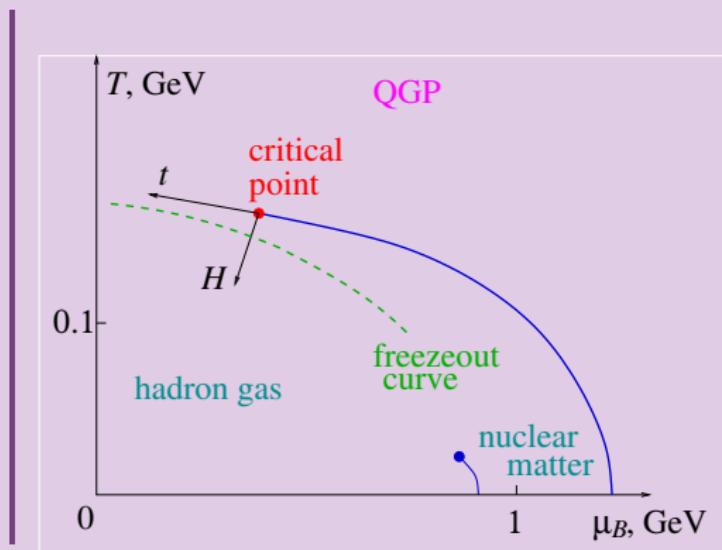
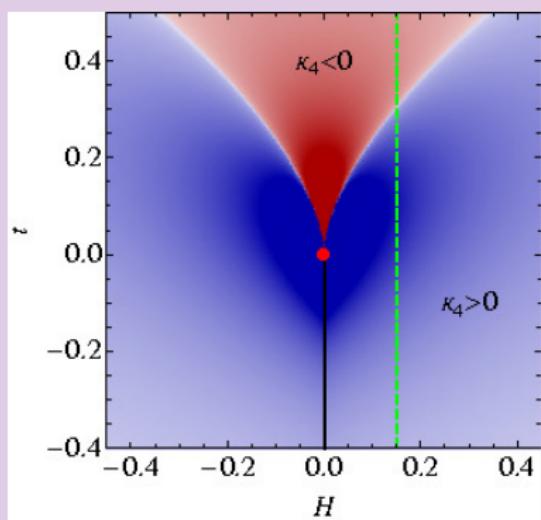
- $m_\pi = 0, \mu_q \neq 0$ : kurtosis **diverges**

$$R_{4,2}^q \sim \left( \frac{\mu_q}{T} \right)^4 / t^{2+\alpha} \quad (t \propto \text{distance to chiral critical line})$$



# SIGN OF KURTOSIS

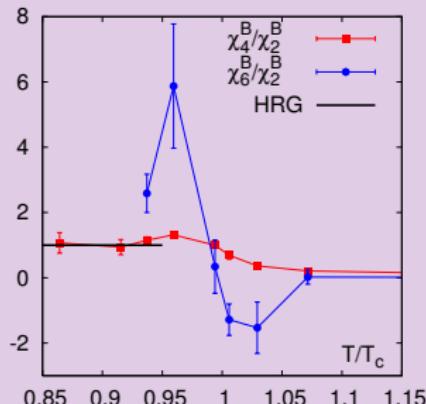
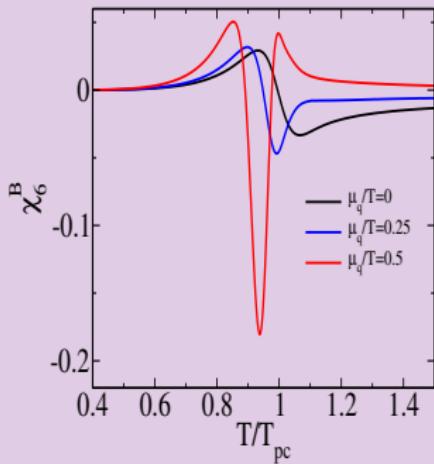
M. Stephanov '11: 3d Ising universality class  $\leadsto$  kurtosis is **negative** close to CEP



# DOES NEGATIVE KURTOSIS SIGNAL CEP?

Negative kurtosis is necessary, but not sufficient condition of CEP.

$$\text{CEP} \rightarrow R_{4,2}^B < 0, \text{ but } R_{4,2}^B < 0 \not\rightarrow \text{CEP}$$



$$\text{sign}(R_{4,2}^B) = \text{sign}\chi_4^B, \quad \chi_4^B(\mu/T) \approx \chi_4^B(0) + \frac{1}{2}\chi_6^B(0) \cdot (\mu/T)^2 + O((\mu/T)^4)$$

$$R_{4,2}^B < 0 \rightarrow \text{non-trivial phase diagram}$$

# MODELING QCD: BEYOND MEAN-FIELD APPROXIMATION

## Functional Renormalization Group

- $p(T, \mu, \textcolor{red}{k})$ ,  $\textcolor{red}{k}$  defines IR cut off  $\sim$   
 $p(T, \mu, \textcolor{red}{k})$  includes modes with momentum  $> \textcolor{red}{k}$ .
- Functional renormalization group equation (exact and general):

$$p(T, \mu, \textcolor{red}{k} - dk) = p(T, \mu, \textcolor{red}{k}) + \boxed{\text{Exact FRG flow}}$$

- Iterating towards  $\textcolor{red}{k} \rightarrow 0$ :  $p(T, \mu, k = 0)$  includes all momentum modes
- Exact FRG is useless, approximations (leading order in gradient expansion):

$$p(T, \mu, \textcolor{red}{k} - dk) = p(T, \mu, \textcolor{red}{k}) + \boxed{\text{Approximate FRG flow}}$$

FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, '02

FRG formulation of PQM model: V. S., B. Stokic, B. Friman & K. Redlich, PRC, '10

# FUNCTIONAL RENORMALIZATION GROUP

The general flow equation for the effective action

$$\partial_k \Gamma_k[\Phi, \psi] = \frac{1}{2} \text{Tr} \left\{ \partial_k R_{kB} \left( \Gamma_k^{(2,0)}[\Phi, \psi] + R_{kB} \right)^{-1} \right\} - \text{Tr} \left\{ \partial_k R_{kF} \left( \Gamma_k^{(0,2)}[\Phi, \psi] + R_{kF} \right)^{-1} \right\}$$

---

The flow equation for the PQM model

$$\begin{aligned} \partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \pi^2]) &= \frac{k^4}{12\pi^2} \left\{ \frac{3}{E_\pi} \left[ 1 + 2n_B(E_\pi; T) \right] + \right. \\ &\quad \left. \frac{1}{E_\sigma} \left[ 1 + 2n_B(E_\sigma; T) \right] - \frac{4N_f N_c}{E_q} \left[ 1 - N(\ell, \ell^*; T, \mu_q) - \bar{N}(\ell, \ell^*; T, \mu_q) \right] \right\} \end{aligned}$$

$n_B(E; T)$  is the boson distribution functions

$N(\ell, \ell^*; T, \mu_q)$  are fermion distribution function modified owing to coupling to gluons

$E_\sigma$  and  $E_\pi$  are the functions of  $k$ ,  $\partial\Omega/\partial\rho$  and  $\rho\partial^2\Omega/\partial\rho^2$

$$E_q = \sqrt{k^2 + 2g\rho}$$

---

FRG defines  $\Omega(k, \rho; T, \mu_Q, \mu_B)$ .

**Physically relevant quantity** is the thermodynamical potential

$\bar{\Omega}(T, \mu_Q, \mu_B) \equiv \Omega(k \rightarrow 0, \rho \rightarrow \rho_0; T, \mu_Q, \mu_B)$ , where  $\rho_0$  is the minimum of  $\Omega$ .